2

Unit

Electric and Magnetic Fields

Charles Coulomb
(1736 – 1806)

Learning Objectives:

After studying this chapter, students should be able to:

- Understand the electric current, electric charges and terms of electrostatics.
- explain the Coulomb's law of electrostatic force, permittivity and forces due to multiple point charges,
- Know about the electric field, electric potential energy.
- Calculate the electric potential at a point and potential difference between two points, electric potential energy.
- Understand the concepts of magnet, magnetic field, magnetic intensity, magnetic lines of force.
- Describe the force on moving charge and force on current carrying conductor placed in a magnetic field.
- Calculate the torque on current carrying conductor in a magnetic field.
- Explain the magnetic dipole and calculate its energy.
- Know about the phenomena of Hall Effect.
- Understand about electromagnetic waves.
- Solve the various numerical problems related with electric and magnetic fields.

21. When a charge moves in a magnetic field, its velocity or K.E. remains unchanged. Explain, why?

Ans: When a charge moves in a magnetic field, it experiences the magnetic Lorentz force which is given by $\vec{F} = q(\vec{V} \times \vec{B})$. This force is perpendicular to both the velocity $(\vec{V})$ of the charge (q) and the magnetic flux density $(\vec{B})$. Hence, the component of force along the direction becomes $\vec{F}\cos 90^o = 0$. Since the force has no effect along the direction of velocity. It has no effect on the magnitude of $\vec{V}$ keeping the K.E. of the charge constant.

22. What is Hall Effect and Hall voltage?

Ans: When a magnetic field is applied to a metallic slab carrying current, an emf is set up across the specimen in the direction perpendicular to both current and the magnetic field. This effect is called Hall Effect. The maximum potential difference across the specimen carrying current inside magnetic field at which the motion of the electrons ceases (electric force is equal to Lorentz force) is called Hall Voltage. When the p.d. reaches the Hall voltage then

Electric Force = Lorentz force

or, $eE = Be\,v$

or, $V_H / d = Bv$

$\therefore \quad V_H = Bvd$

Where, E is the electric field produced due to Hall voltage, v is the drift velocity of electron and d is the separation between opposite faces between which Hall voltage is produced.

23. A charge particle does not gain kinetic energy when passing through a magnetic field but it gains kinetic energy when passing through an electric field. Explain why?

OR **If a magnetic force does not work charge particles, how can have any effect on the particle's motion?**

Ans: When a charge particle passes through an electric field, the displacement of the charge is in the direction of the force on the charge due to the electric field. As a result, work is done by the electric field on the charge. Since work is done on the charge, the kinetic energy of the charge increases. However, when the charge particle passes through a magnetic field, the displacement of the charge is always perpendicular to the force on it due to the magnetic field. Since the force and the displacement are perpendicular to each other, the work done by the force is zero (as $W = F.\,l = F\,l\cos\theta = Fl \times \cos 90^o = 0$). Since, no work is done on the charge, the, kinetic energy of the charge does not change.

Worked Out Examples

1. A charge $q_1 = 3 \times 10^{-6}$ c is located at the origin of the x – axis. A second charge $q_2 = -5 \times 10^{-6}$ c is also on the x – axis 4 m from the origin in the positive x-direction (a) calculate the electric field at the mid-point p of the line joining the two charges (b) at what point p' on the line is the resultant field zero? ***[TU Microsyllabus 2074,W; 14.1]***

Solution:

Charge located at origin $q_1 = 3 \times 10^{-6}$ C

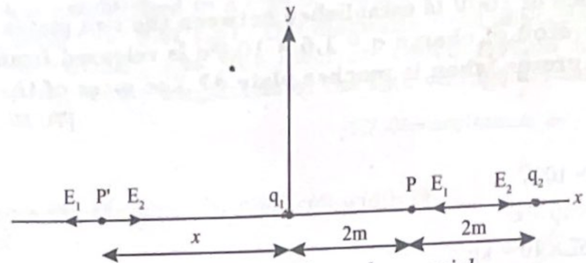
Second charge $q_2 = -5 \times 10^{-6}$ C

Distance from q_1 to p point = 2m

Distance from p to q_2 = 2m

Distance from q_1 to p' = x = ?

The electric field at the mid-point p of line joining q_1 and q_2 = ?



[Fig. 25: Charges located on x-axis]

a. The electric field E_1 at a point p due the charge q_1 is

$$E_1 = \frac{q_1}{4\pi\varepsilon_0(2)^2} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{4} = 6.75 \times 10^3 \text{ N/C toward the positive x-axis.}$$

Again, electric field E_2 at a point P due to the charge q_2 is,

$$E_2 = \frac{q_2}{4\pi\varepsilon_0(2)^2} = \frac{9 \times 10^9 \times -5 \times 10^{-6}}{4} = -11.25 \times 10^3 \text{ N/C.}$$

$\therefore$ $E_2 = 11.25 \times 10^3$ N/C which is along same direction of E_1

The positive sign means field direction along +ve x-axis.

The fields E_1 and E_2 and directed along the same direction

So, the resultant electric field at p will be

$E = E_1 + E_2 = 6.75 \times 10^3 \text{ N/C} + 11.25 \times 10^3 \text{ N/C}$

$\therefore$ $E = 18 \times 10^3$ N/C.

Hence, electric field at p due to charges q_1 q_2 is 18×10^3 N/C.

b. The electric field at a point p' due to the charge q_1 and q_2 are $E_1 = \frac{q_1}{4\pi\epsilon_o x^2}$ and $E_2 = \frac{q_2}{4\pi\epsilon_o(x+4)^2}$

The resultant field at p' will be zero. So

$\therefore$ $E = E_1 + E_2 = 0$

or, $E_1 = -E_2$

or, $|E_1| = |E_2|$

or, $\frac{q_1}{4\pi\epsilon_o x^2} = \frac{q_2}{4\pi\epsilon_o(x+4)^2}$

or, $3(x+4)^2 = 5x^2$

or, $2x^2 - 24x - 48 = 0$

or, $x^2 - 12x - 24 = 0$

or, $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4.1(-24)}}{2}$

$\therefore$ $x = 13.75$ m and $x = -1.75$m

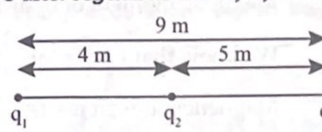
Therefore, E = 0 at x = 13.75m towards left from the origin but x = – 1.75m represents a point between the charges at which $E_1 = E_2$. However, as indicated case (a) since, between the charges, E_1 and E_2 have the same direction and consequently the resultant field is not zero.

2. **Three charges $q_1 = 3 \times 10^{-6}$c, $q_2 = -5 \times 10^{-6}$ c and $q_3 = -8 \times 10^{-6}$ c are positioned on a straight line. Find the potential energy of the charges.** ***[TU Microsyllabus 2074,W; 14.2]***

Solution:

We have given charges $q_1 = 3 \times 10^{-6}$ C, $q_2 = -5 \times 10^{-6}$ C and $q_3 = -8 \times 10^{-6}$ C

In figure 26 potential energy due to q_1q_2, q_1q_3 and q_2q_3 are obtained by



[Fig. 26: Location of three charges]

$$E_p = \frac{1}{4\pi\epsilon_o}\left[\frac{q_1q_2}{4} + \frac{q_1q_3}{9} + \frac{q_2q_3}{5}\right]$$

$$E_p = 9 \times 10^9 \left[\frac{(3 \times 10^{-6}) \times (-5 \times 10^{-6})}{4} + \frac{(3 \times 10^{-6})(-8 \times 10^{-6})}{9} + \frac{(-5 \times 10^{-6}) \times (-8 \times 10^{-6})}{5}\right]$$

$\therefore$ $E_p = 1.43 \times 10^{-2}$ J.

Hence, required potential energy of the charges is 1.43×10^{-2}J.

3. A potential difference of 100V is established between the two plates A and B. plate B being high potential. A proton of charge q = 1.6 × 10^{-19}c is released from plate B. What will be the velocity of the proton when it reaches plate A? The mass of the proton is 1.67 × 10^{-27} kg. *[TU Microsyllabus 2074,W; 14.3]*

Solution:

Potential difference V = 100V

Proton charge q = 1.6 × 10^{-19} c

Mass of proton m = 1.67 × 10^{-27} kg

We know that,

$$E_k = qV$$

$$\frac{1}{2} m v_A^2 = qV$$

$$v_A = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 100}{1.67 \times 10^{-27}}} = 1.38 \times 10^5 \text{ m/sec}$$

Thus, required velocity of the proton = 1.38 × 10^5 m/sec

4. Suppose a copper wire carries 10A of current and has a cross-section of 10^{-6} m², each atom of copper contributes one electron that is free to move. So the electron carrier density N_n is about the same as the density of atoms which is about 7 × 10^{28} atom per m³. The charge on an electron is +1.6 × 10^{-19} (a) what is the drift velocity of the electrons? (b) How long would it take an electron to move from one terminal of a battery to the other if this wire were 1 m long?

Solution:

Current I = 10A. Cross – section area A = 10^{-6}m². Electron carrier density N_n = 7 × 10^{28} atom/m³.

Charge on an electron (q) = 1.6 × 10^{-19}c Drift velocity V_d = ? Time taken by electron t = ?

a. We have the relation $V_d = \frac{I}{AqN_n}$

$$\therefore \quad V_d = \frac{10}{10^{-6} \times 1.6 \times 10^{-19} \times 7 \times 10^{28}} = 9 \times 10^{-4} \text{ m/sec}$$

b. Time taken $t = \frac{x}{V_d} = \frac{1}{9 \times 10^{-4}} = 1.1 \times 10^3$ sec = 11 mins.

Hence, required drift velocity and time taken by electron are 9 × 10^{-4} m/sec and 11 min respectively.

5. Assume that the electron in a hydrogen atom is essentially in a circular orbit of radius 0.5×10^{-10} m, and rotates about the nucleus at the rate of 10^{14} times per second. What is the magnetic moment of the Hydrogen atom due to the orbital motion of the electron? *[TU Microsyllabus 2074, W; 16.1]*

Solution:

Radius of circular orbit r = 0.5 × 10^{-10} m Frequency f = 10^{14}sec^{-1}

Magnetic moment of hydrogen μ = ?

We know that $I = \frac{q}{t} = qf$. Since, $\frac{1}{t} = f$

Magnetic moment μ = IA

$$\mu = I.\ \pi r^2$$

$$\mu = qf\ \pi r^2$$

$$\mu = 1.6 \times 10^{-19} \times 10^{14} \times \pi \times (0.5 \times 10^{-10})^2$$

$$\therefore \quad \mu = 1.26 \times 10^{-25} \text{ m}^2$$

Hence, the required magnetic moment of hydrogen atom is 1.26 × 10^{-25} Am². It means, hydrogen atom is essentially a small bar magnet and will behave as such in a magnetic field.

6. **The current of 50A is established in a slab of copper 0.5 cm thick and 2 cm wide. The slab is placed in a magnetic field B of 1.5T. The magnetic field is perpendicular to the plane of the slab and to the current. The free electron concentration in copper is 8.4 × 10^{28} electrons/m^3. What will be the magnitude of the Hall voltage across the width of the slab?**

[TU Microsyllabus, 2074, W; 16.2, TU Exam 2074]

Solution:

Current (I) = 50A, thickness of copper (x) = 0.5 cm, width of copper slab (b) = 2 cm

Magnetic field (B) = 1.5T

Free electrons concentration (N) = 8.4 × 10^{28} electrons/m^3.

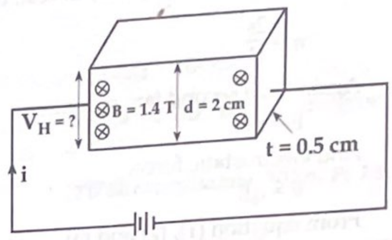
Magnitude of hall voltage (V_H) = ?

We know that $V_H = \frac{IB\,b}{NqA}$

$$V_H = \frac{50 \times 1.5 \times 2 \times 10^{-2}}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-4}}$$

$\therefore$ $V_H = 1.12 \times 10^{-6}$ V

Hence, required hall voltage is 1.12×10^{-6} V.



[Fig. 27: Representation of Hall Effect due to charge particle in a magnetic field with in the slab of copper]

7. **Four charges of equal magnitude are placed at the corners of a square as shown in figure 28. What is the electric field at the centre of the square, point O?**

[TU Microsyllabus 2074, P; 14.6]

Solution:

Here, four charges of equal magnitude

$q_1 = +3 \times 10^{6}$ C

$q_2 = -3 \times 10^{-6}$ C

$q_3 = -3 \times 10^{-6}$ C

$q_4 = +3 \times 10^{-6}$ C

Consider PQRS be given square then PQ = PS = QR = RS = 0.23 m

From figure 28, we get

$PR = \sqrt{0.25^2 + 0.25^2} = 0.354$ m

$\therefore$ PO = 0.176 m = QO = SO = RO

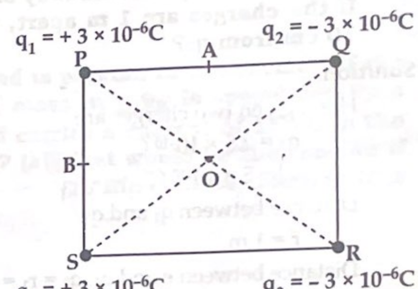
The total electric field at the centre of the square,

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{PO^2} + \frac{q_2}{QO^2} + \frac{q_3}{RO^2} + \frac{q_4}{SO^2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3 \times 10^{-6}}{0.25^2} + \frac{(-3 \times 10^{-6})}{0.25^2} + \frac{(-3 \times 10^{-6})}{0.25^2} + \frac{(3 \times 10^{-6})}{0.25^2}\right]$$

$E = 0\ NC^{-1}$

Hence, required the electric field at the centre of the square at point O is zero.



[Fig 28: Four charges at the corners of a square having equal magnitude]

8. **Two large parallel plates are separated by a distance of 5 cm. The plates have equal but opposite charges that create and electric field in the region between the plates. An α-particle (q = 3.2 × 10^{-19} C, m = 6.68 × 10^{-27} kg) is released from the positively charged plate and it strikes the negative y charged plate 2 × 10^{-6} sec later. Assuming that the electric field between the plates is uniform and perpendicular to the plates, what is the strength of the electric field?** ***[TU Microsyllabus 2074; P; 14.8, TU Model question 2074]***

Solution:

Here, distance between plates (s) = 5 cm = 5×10^{-2} m

Charge of α-particle (q) = 3.2×10^{-19} c

Mass of α-particle (m) = 6.68×10^{-27} kg

Time taken (t) = 2×10^{-6} sec

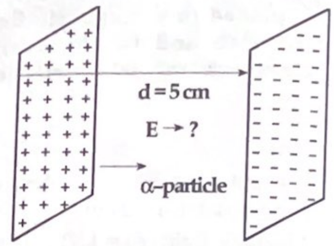
Electric field between the plates (E) = ?

We know that from equation of motion,

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}at^2 \quad [\because \text{Initially at rest, } u = 0]$$

$$a = \frac{2s}{t^2} \quad \ldots(1)$$



[Fig. 29: Two parallel plates having opposite charges at which α–particle released from +ve plate to –ve]

Again from second law of motion

$$F = ma \quad \ldots(2)$$

And electrostatic force,

$$F = qE \quad \ldots(3)$$

From equation (1), (2) and (3)

$$ma = qE$$

$$m\frac{2s}{t^2} = qE$$

$$E = \frac{2ms}{qt^2} = \frac{2 \times 6.68 \times 10^{-27} \times 5 \times 10^{-2}}{3.2 \times 10^{-19} \times (2 \times 10^{-6})^2} = 521.875$$

$$\therefore \quad E \approx 522\ NC^{-1}$$

Hence, strength of field between the plate is 522 NC^{-1}.

9. An electron is placed midway between two fixed charges, $q_1 = 2.5 \times 10^{-10}$ C and $q_2 = 5 \times 10^{-10}$ C. If the charges are 1 m apart, what is the velocity of the electron when it reaches a point 10 cm from q_2? ***[TU Microsyllabus 2074, P; 14.2]***

Solution:

Here, given two charges are

$$q_1 = 2.5 \times 10^{-10}\ C$$

$$q_2 = 5 \times 10^{-10}\ C$$

Distance between q_1 and q_2

$$r = 1\ m$$

Distance between e and q_1, $q_2 = r_1 = r_2 = 0.5$ m

Distance travelled by e^- towards q_2 from initial position

$$S = 0.5\ m - 0.1\ m = 0.4\ m$$

Now we know that

From equation of motion

$$v^2 = u^2 + 2as \quad \text{Since, } u = 0\ ms^{-1}$$

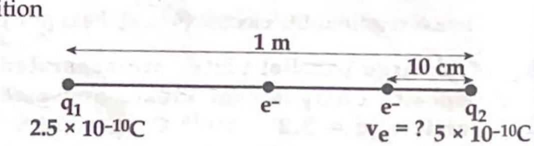
$$v = \sqrt{2as} \quad \ldots(1)$$

Again, Electrostatic force between q_1 and e at rest condition

$$F_1 = \frac{q_1 e}{4\pi \epsilon_0 r_1^2}$$

Electrostatic force between q_2 and e

$$F_2 = \frac{q_2 e}{4\pi \epsilon_0 r_2^2}$$



[Fig. 30: Representation of electron in between two charges]

Now, Net force $F = F_2 - F_1 = \frac{q_1 e}{4\pi \epsilon_0 r_1^2} - \frac{q_2 e}{4\pi \epsilon_0 r_2^2}$

$$F = \frac{e}{4\pi\epsilon_0} \left[\frac{q_2}{r_2^2} - \frac{q_1}{r_1^2}\right] = \frac{1.6 \times 10^{-19} \times 9 \times 10^9}{(0.5)^2} \times [5 \times 10^{-10} - 2.5 \times 10^{-10}]$$

$$= 1.44 \times 10^{-18}\ N$$

From Newton's second law of motion

$F = ma = 1.44 \times 10^{-18}$

or, $a = \dfrac{1.44 \times 10^{-19}}{9.1 \times 10^{-31}}$ [$\because m = m_e = 9.1 \times 10^{-31}$ kg]

$\therefore$ $a = 1.58 \times 10^{12}\ ms^{-2}$

Substituting value of 'a' in equation (1), we get

$v = \sqrt{2 \times 1.58 \times 10^{12} \times 0.4}$

$\therefore v = 1.125 \times 10^{6}\ ms^{-1}$.

Hence, required velocity of electron when it reaches a point 10 cm from q_2 is $1.125 \times 10^{6}\ ms^{-1}$.

10. What force is experienced by a wire of length l = 0.08 m at an angle of 20° to the magnetic field direction carrying a current of 2A in a magnitude field 1.4 T ?

[TU Microsyllabus 2074, P; 16.1]

Solution:

Given, length of wire (l) = 0.08 m

Angle between B and l is (θ) = 20°

Magnetic field (B) = 1.4 T

Current (I) = 2 A

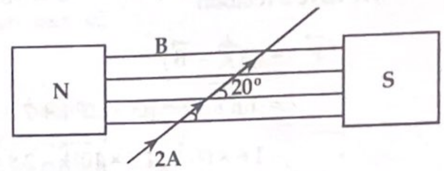
Force experienced by a wire (F) = ?

We know that,

$F = BIl \sin\theta$

$= 1.4 \times 2 \times 0.08 \times \sin 20°$

$F = 7.66 \times 10^{-2}$ N



[Fig. 31: Current carrying wire in uniform magnetic field at angle 20°]

Hence, required force experienced by wire is 7.66×10^{-2} N.

11. The earth's magnetic field at the equator is 4×10^{-5} T and is parallel to the surface of the earth in the south-north direction. A wire 2 m long of mass m = 9g is suspended by a string. The wire is also parallel to the earth's surface and carries a current of 150 A in the east-west direction. (a) What is the tension of the string? (b) What would be the tension if the current was in the west-east direction? ***[TU Microsyllabus 2074, P; 16.2]***

Solution:

Here is given, Earth magnetic field at equator (B) = 4×10^{-5} T

Length of wire (ℓ) = 2m

Mass of wire (m) = 9 gm = 9×10^{-3} kg

Current carried by wire (I) = 150 A

a. Tension on the string (T_{NS}) = ?

b. Tension on the string when current was in the west-east direction (T_{WE}) = ?

For (a)

In 1st case tension on the string or wire is provided by weight and magnetic fore.

i.e.,

$T = mg + BI\ell$

$= 9 \times 10^{-3} \times 9.8 + 4 \times 10^{-5} \times 150 \times 2$

$= 0.1002$ N

$= 10.02 \times 10^{-2}$ N

For (b)

$T' = mg - BI\ell$

$= 9 \times 10^{-3} \times 9.8 - 4 \times 10^{-5} \times 150 \times 2$

$= 0.0762$ Since, $g = 9.8\ ms^{-2}$

$= 7.62 \times 10^{-2}$ N.

Hence, required tension on the string when it is at north-south direction and west-east direction are 10.02×10^{-2} N and 7.62×10^{-2} N respectively.

12. A proton is moving with a velocity $\vec{v} = (3 \times 10^5\ \hat{i} + 7 \times 10^5\ \hat{k})\ ms^{-1}$ in a region where there is a magnetic filed $\vec{B} = 0.4\hat{j}T$. What is the force experienced by the proton?

[TU Microsyllabus 2074, P; 16.12]

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Solution:

Velocity of proton $(\vec{v}) = (3 \times 10^5\ \hat{i} + 7 \times 10^5\ \hat{k})\ ms^{-1}$

Magnetic field $(\vec{B}) = 0.4\hat{j}T$

We know, mass of proton $(m_p) = 1.67 \times 10^{-27}$ kg

Charge of proton (q) = 1.6×10^{-19} c

Force experienced by the proton $(\vec{F})$ = ?

We have a relation

$$\vec{F} = q(\vec{v} \times \vec{B})$$
$$= 1.6 \times 10^{-19} [(3 \times 10^5\ \hat{i} + 7 \times 10^5\ \hat{k}) \times 0.4\hat{j}]$$
$$= 1.6 \times 00^{-19} [1.2 \times 10^5 \hat{k} - 2.8 \times 10^5 \hat{i}]$$
$$= 1.6 \times 10^{-19} [1.2\hat{k} - 2.8\hat{i}] \times 10^5$$
$$\vec{F} = (1.92\ \hat{k} - 4.48\ \hat{i}] \times 10^{-14} N$$

Hence, force experienced by the proton is $(1.92\ \hat{k} - 4.48\ \hat{i}] \times 10^{-14}$ N.

13. A proton is accelerated through a potential difference of 200 V. It then enters a region where there is a magnetic filed B = 0.5 T. The magnetic field is perpendicular to the direction of motion of the proton. What is the force experienced by the proton?

[TU Microsyllabus 2074, P; 16.13]

Solution:

Here, potential difference (p.d.), V = 200 V

Magnetic field (B) = 0.5 T

Force experienced by the proton (F) = ?

We know that, at equilibrium condition, K.E. = P.E.

$$\frac{1}{2} m_p v_p^2 = qV$$
$$v_p^2 = \sqrt{\frac{2qV}{m_p}}$$
$$= \left(\frac{2 \times 1.6 \times 10^{-19} \times 200}{1.67 \times 10^{-27}}\right)^{1/2}$$

Since, mass of proton $m_p = 1.67 \times 10^{-27}$ kg, $q = 1.6 \times 10^{-19}$ C

$$= 1.957 \times 10^5\ ms^{-1}$$
$$\approx 1.96 \times 10^5\ ms^{-1}$$

Again, we know Lorentz force,

$$F = Bqv_p$$

Since, $v_p \perp^r B$

$$= 0.5 \times 1.6 \times 10^{-19} \times 1.96 \times 10^5$$
$$\therefore F = 1.568 \times 10^{-14} N$$

Hence, required force experienced by the proton is 1.568×10^{-14} N.